SINGLE STROKE

Math Circle @ IIT Palakkad
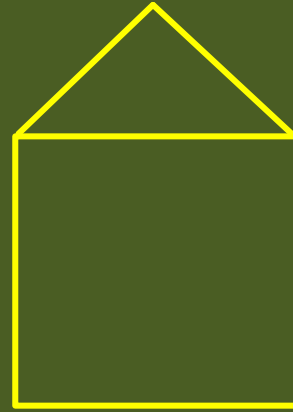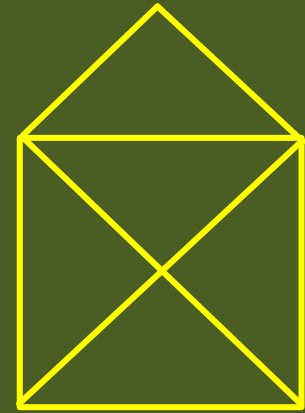
9/Nov/2024

SINGLE STROKE

Draw the given picture in a single stroke.
That is:

1. Do not lift the pencil from the paper
2. Do not double trace
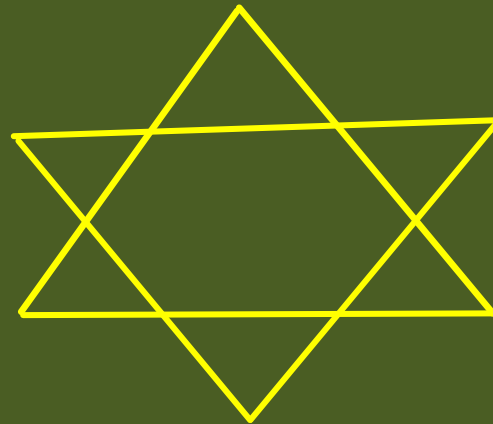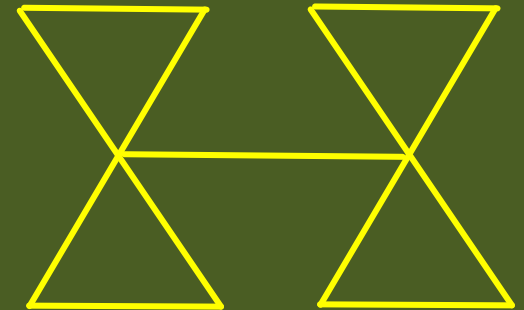3. You can cross through
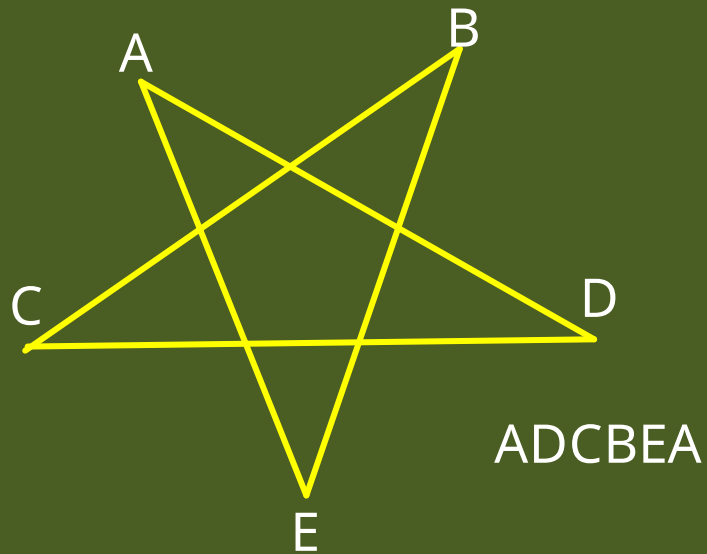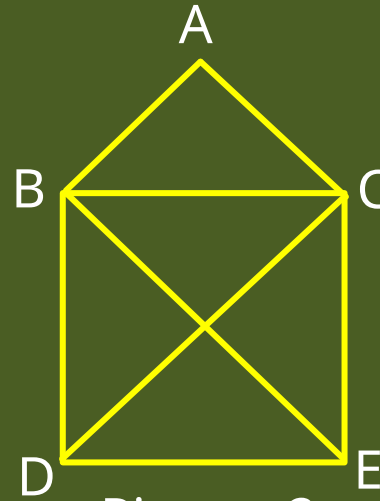
Picture 1

Picture 2

Picture 3

Picture 4

Picture 5

READ your STROKE

1. Give names to corners and crossings (if needed)
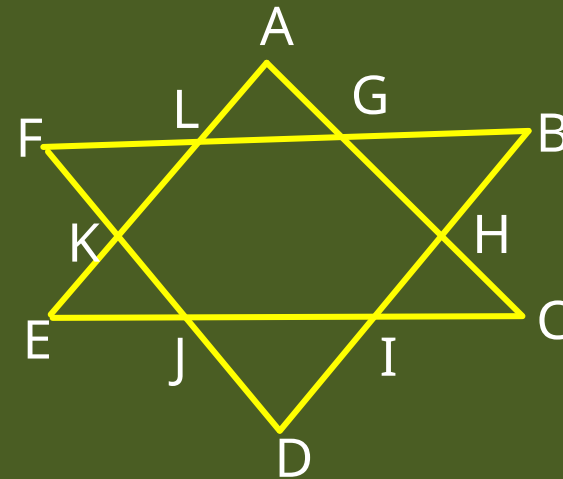2. Read out the names in order

Picture 2

DCBEDBACE

DECBDCABE

ADCBEA

Picture 3

Picture 4

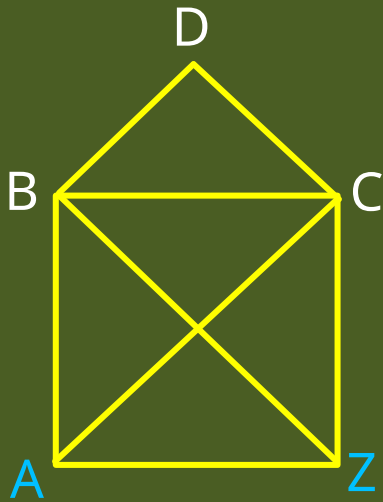AGBHIDJKFLG

HCIJEKLA

GHIKLGBHCI

DJEKFLAG

SINGLE A-Z STROKE

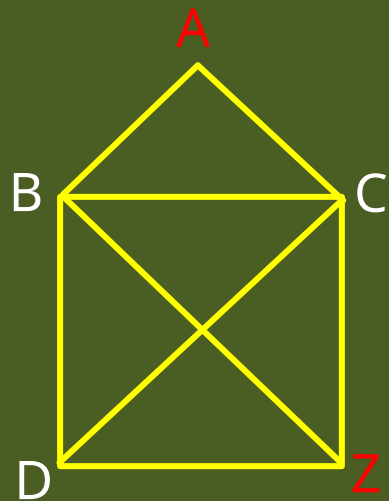Draw the given picture in a single stroke starting at A and ending at Z

Old rules still apply:
1. Do not lift the pencil from the paper
2. Do not double trace
3. You can cross through

Picture 5

Picture 5 Dobara

Picture 2

Picture 2 Dobara
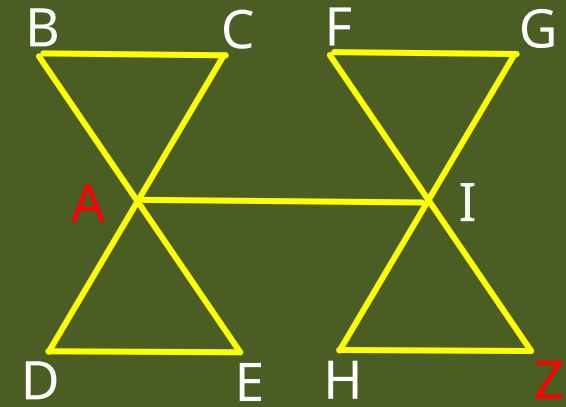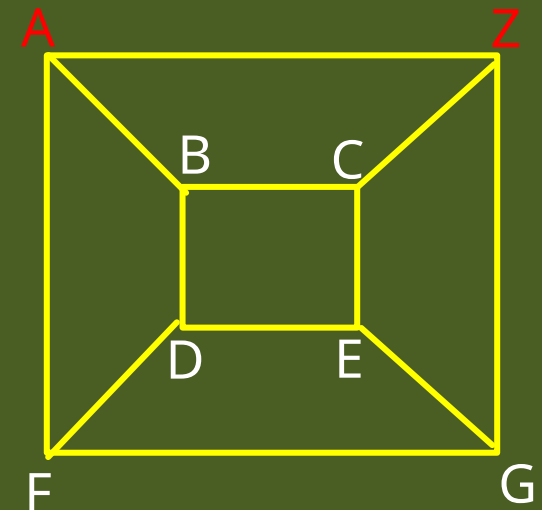
Picture 3

Picture 6

WHY DID WE FAIL?
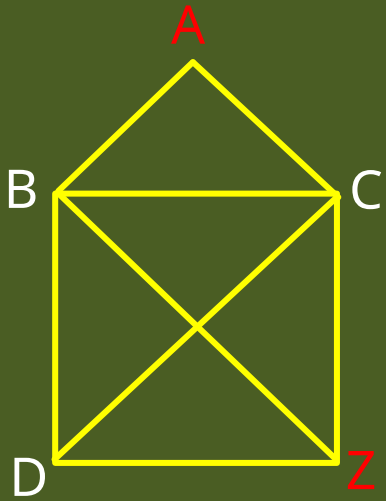
Picture 2
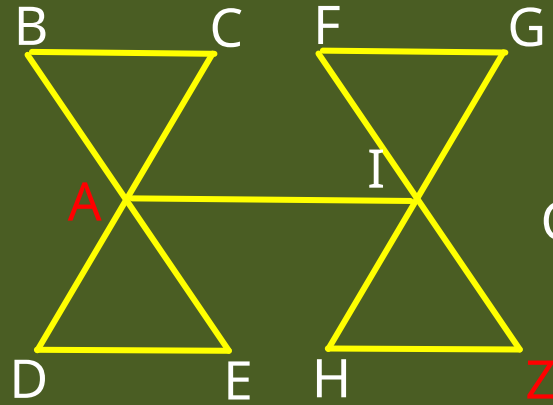
Picture 5
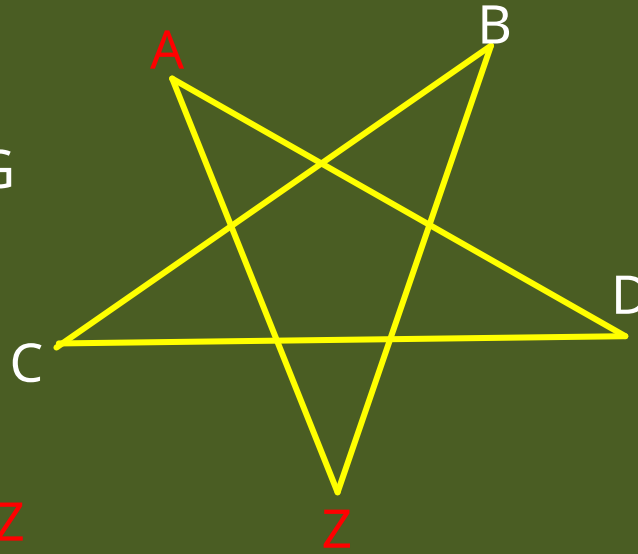
Picture 3
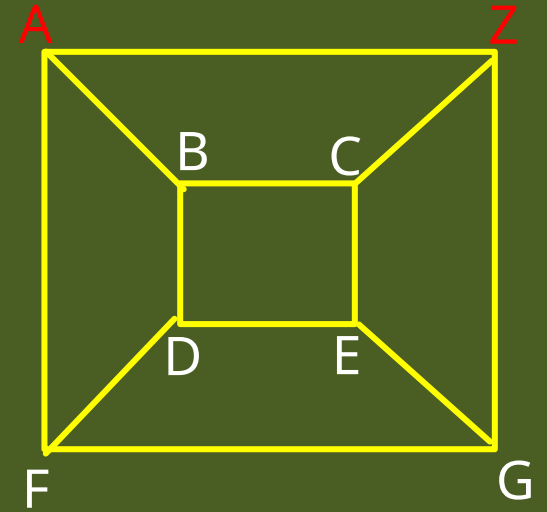
Picture 6

We cannot trace Picture ___ in a single stroke from A to Z because _____

_____

_____

WHEN WILL WE FAIL?

We cannot draw the given picture in a single stroke from A to Z when

1. _____

2. _____

3. _____

4. _____
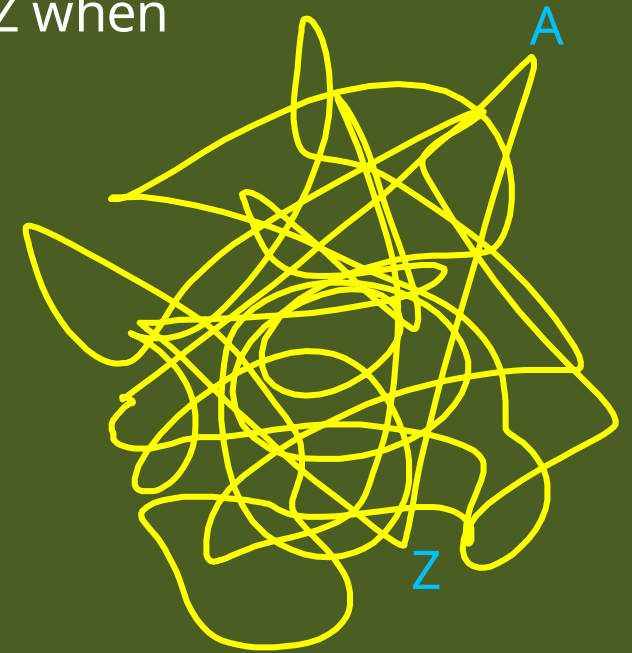
Given Picture

FOUR NEW WORDS

A

B   C

D   Z

GRAPH 2

B
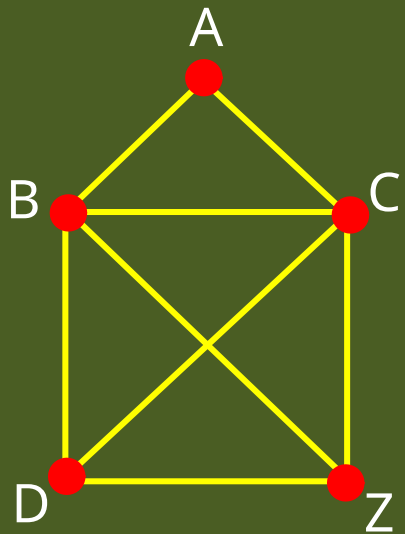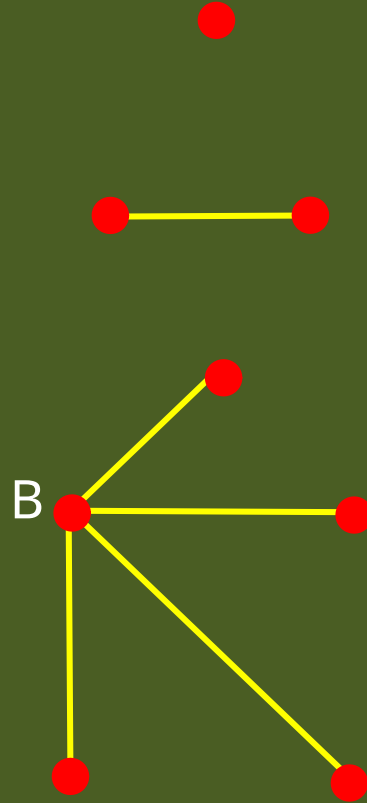
1. VERTEX: Named corner or crossing.
(Eg: A, B, Z)

2. EDGE: Segment from
one corner to next
(Eg: AB, AC, BZ)

3. DEGREE: Number of segments
touching a vertex
is called its degree
(Eg: Degree of B is 4
Degree of D is 3)

WHEN WILL WE FAIL?

We cannot draw the given graph in a single stroke from A to Z when
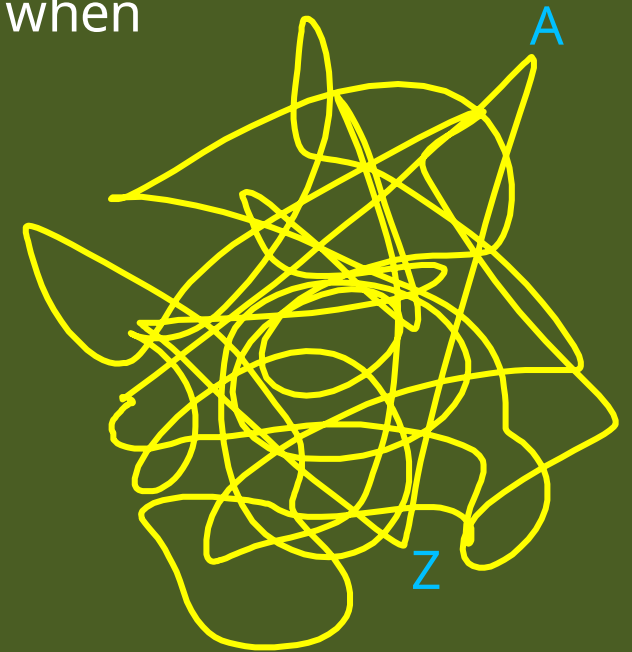
1. _____

2. _____

3. _____

4. _____

Given Graph

Vertex: Named corner or crossing.

Edge: Segment from one corner to next

Degree of a vertex: Number of segments touching that vertex

WHEN WILL WE FAIL?

We cannot draw the given graph in a single stroke from A to Z when

1. Vertex A has even degree

2. Vertex Z has even degree

3. Any other vertex has odd degree

4 . The graph has multiple pieces

Vertex: Named corner or crossing.

Edge: Segment from one corner to next

Degree of a vertex: Number of segments touching that vertex

Given Graph

WHY DO WE FAIL WHEN WE FAIL?

We cannot draw the given graph in a single stroke from A to Z when

1. Vertex A has even degree because _____
   _____

2. Vertex Z has even degree because _____
   _____

3. Any other vertex has odd degree because _____
   _____

4 . The graph has multiple pieces because _____
   _____

Given Graph

Vertex: Named corner or crossing.
Edge: Segment from one corner to next
Degree of a vertex: Number of segments touching that vertex

WHY DO WE FAIL WHEN WE FAIL?

We cannot trace the given graph in a single stroke from A to Z when vertex A has even degree because

1. _____

2. _____

3. _____

4. _____

A

Vertex: Named corner or crossing.

Edge: Segment from one corner to next

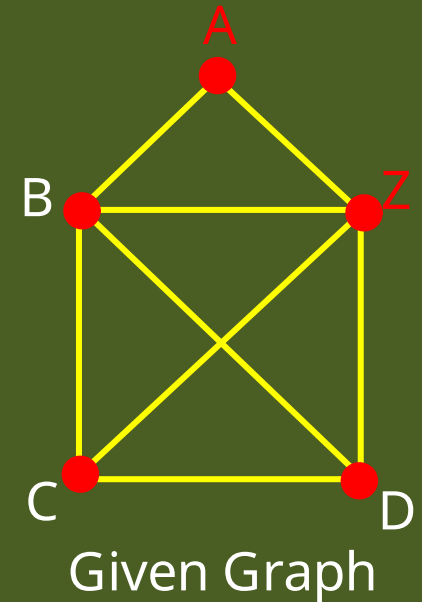Degree of a vertex: Number of segments touching that vertex
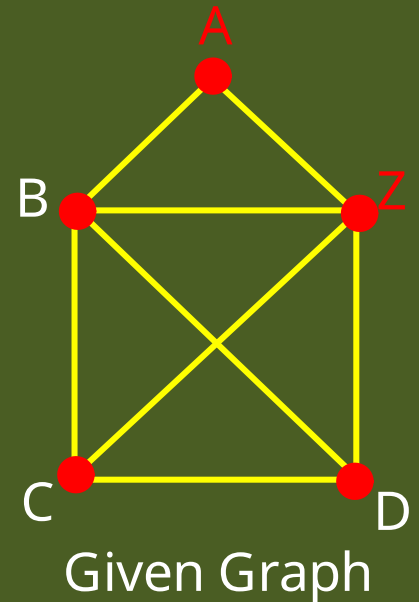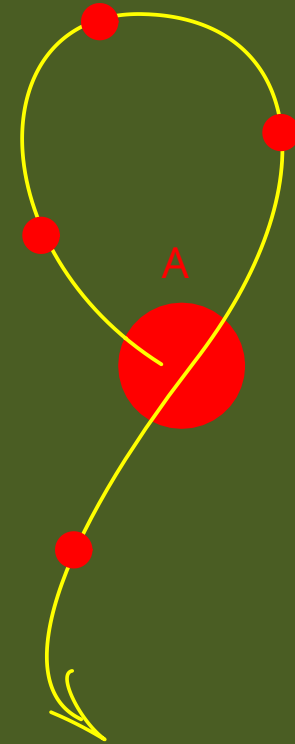
WHY DO WE FAIL WHEN WE FAIL?

We cannot trace the given graph in a single stroke from A to Z when vertex A has even degree because

1. We start tracing one segment touching A
2. Whenever we pass through A again we trace two segments touching A
3. When we finish tracing we will have traced an odd number of segments touching A
4. But A touches an even number of segments

WHY DO WE FAIL WHEN WE FAIL?

We cannot trace the given graph in a single stroke from
A to Z when vertex Z has even degree because

1. _____

2. _____

3. _____

4. _____

Z

WHY DO WE FAIL WHEN WE FAIL?
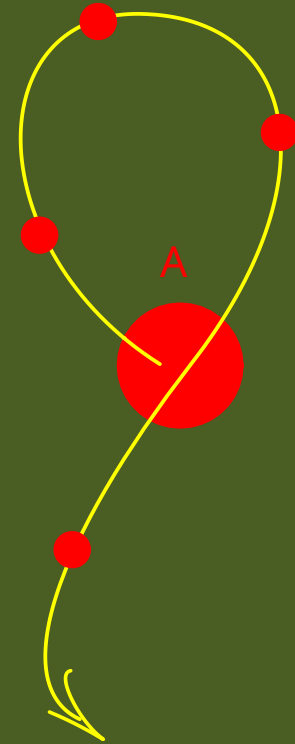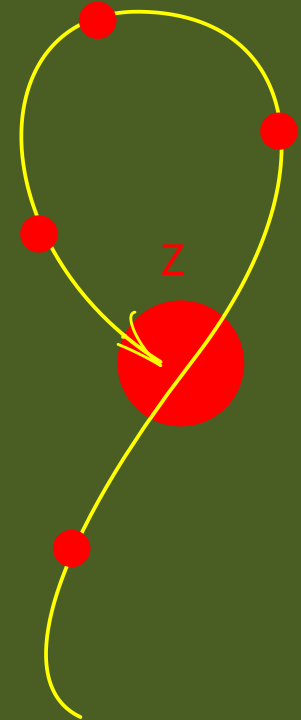
We cannot trace the given graph in a single stroke from A to Z when vertex Z has even degree because

1. We end tracing with one segment touching Z
2. Whenever we pass through Z before we trace two segments touching Z
3. When we finish tracing we will have traced an odd number of segments touching Z
4. But Z touches an even number of segments

WHY DO WE FAIL WHEN WE FAIL?

We cannot trace the given graph in a single stroke from
A to Z when any other vertex has odd degree because

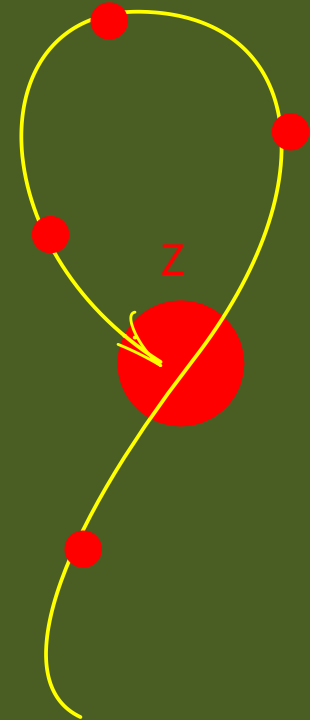1. _____

2. _____

3. _____

4. _____

B

WHY DO WE FAIL WHEN WE FAIL?

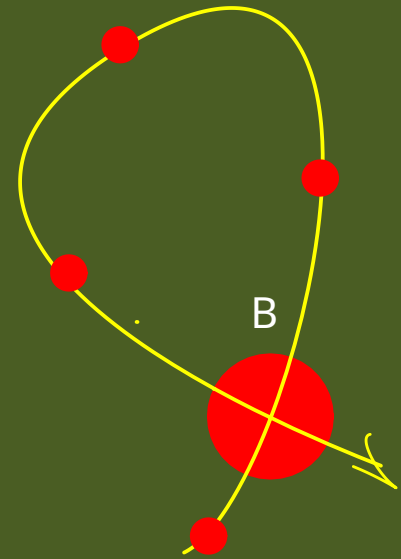We cannot trace the given graph in a single stroke from A to Z when any other vertex has odd degree because

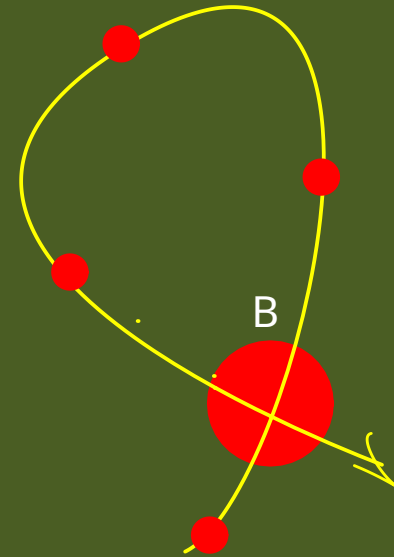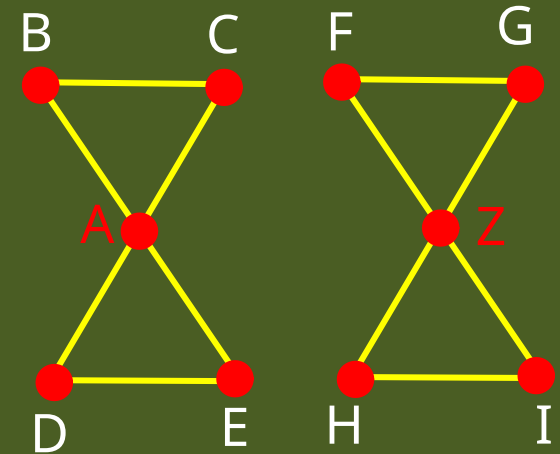1. Let B be a vertex other than A or Z
2. Whenever we pass through B we trace two segments touching Z
3. When we finish tracing we will have traced an even number of segments touching B
4. But B touches an odd number of segments

B

WHY DO WE FAIL WHEN WE FAIL?

We cannot trace the given graph in a single stroke from A to Z when the graph has multiple pieces because
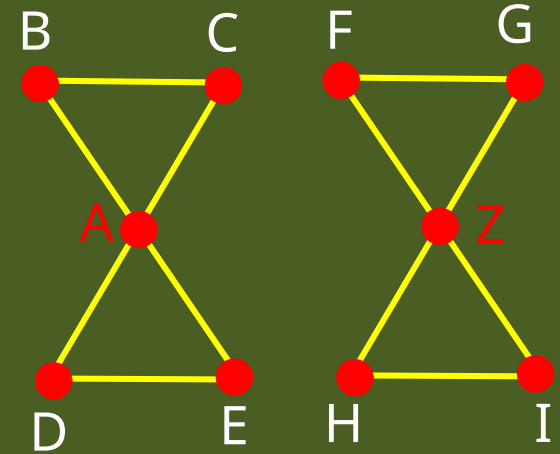
1. _____

2. _____

3. _____

4. _____

WHY DO WE FAIL WHEN WE FAIL?

We cannot trace the given graph in a single stroke from A to Z when the graph has multiple pieces because
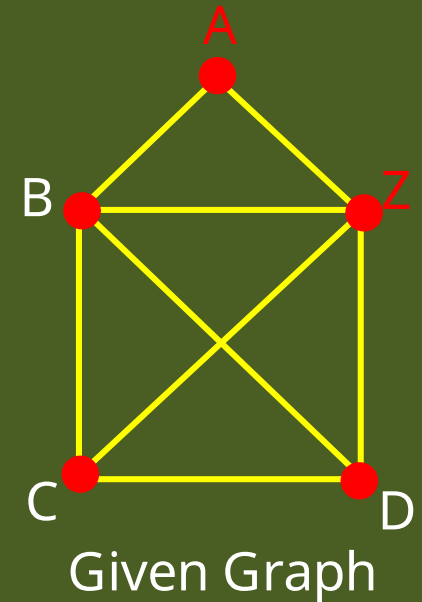
1. We cannot jump from one piece to another without lifting the pencil from the paper

WHEN WILL WE FAIL?

We cannot draw the given graph in a single stroke from A to Z when

1. Vertex A has even degree

2. Vertex Z has even degree

3. Any other vertex has odd degree

4 . The graph has multiple pieces
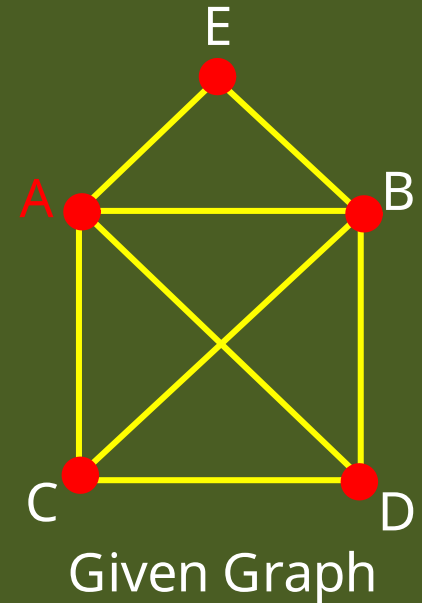
AND WE KNOW WHY

Given Graph

SINGLE A-A STROKE

We cannot draw the given graph in a single stroke from A to A when

1. _____ because _____

   _____

2 . _____ because _____

   _____

E

A   B

C   D

Given Graph
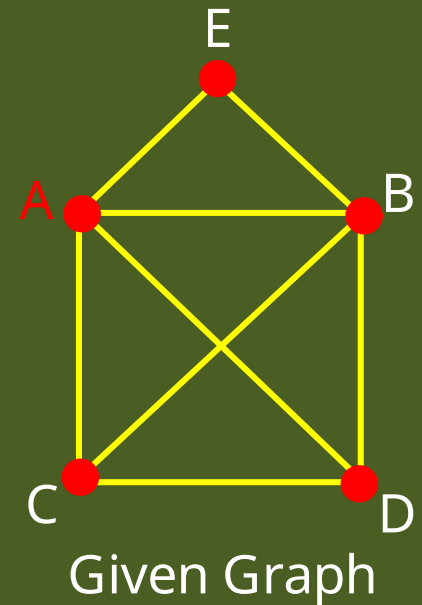
SINGLE A-A STROKE

We cannot draw the given graph in a single stroke from A to A when

1. Any vertex has an odd degree

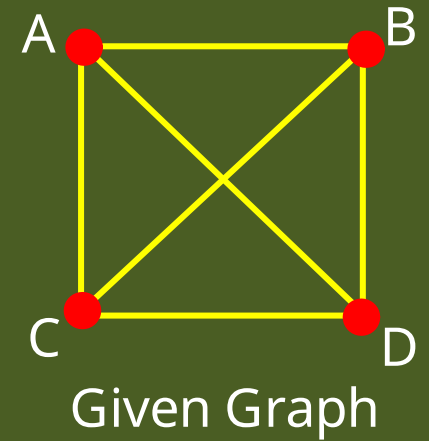2. The graph has multiple pieces

Given Graph

AND WE KNOW WHY

SINGLE STROKE

We cannot draw the given graph in a single stroke from any vertex to any vertex when

1. _____ because _____
   _____

2. _____ because _____
   _____

Given Graph

SINGLE STROKE

We cannot draw the given graph in a single stroke from any vertex to any vertex when

1. The number of odd degree vertices is neither 0 or 2 because
   (a) Any A-Z single stroke traces an even number of segments touching every vertex other vertex and and odd number of segments touching A and Z
   (b) Any A-A single stroke traces an even number of segments touching every vertex

2. If the graph has more than one piece because we cannot jump from one piece to another without lifting the pencil from the paper
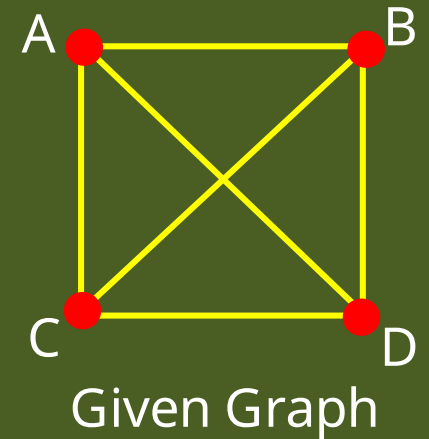
Given Graph

FOUR GAMES

1. SINGLE A-Z STROKE

2. SINGLE A-A STROKE

3. SINGLE STROKE

4. SINGLE CLOSED STROKE

Which is your favourite?

IIT PALAKKAD

SINGLE CLOSED STROKE

We cannot draw the given graph in a single closed stroke when

1. Any vertex has an odd degree

2 . The graph has multiple pieces

Can we draw any given graph in a single closed stroke provided

1. **Every vertex has an even degree**
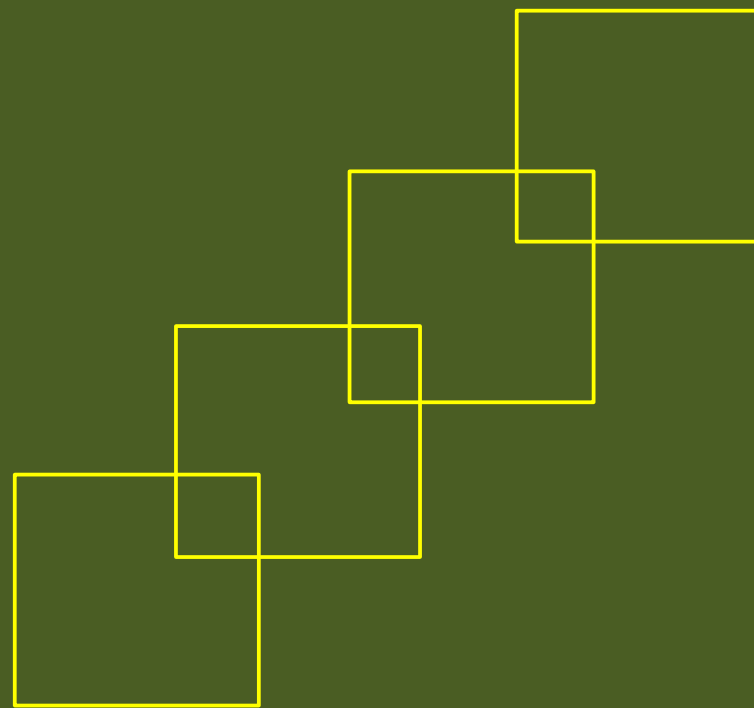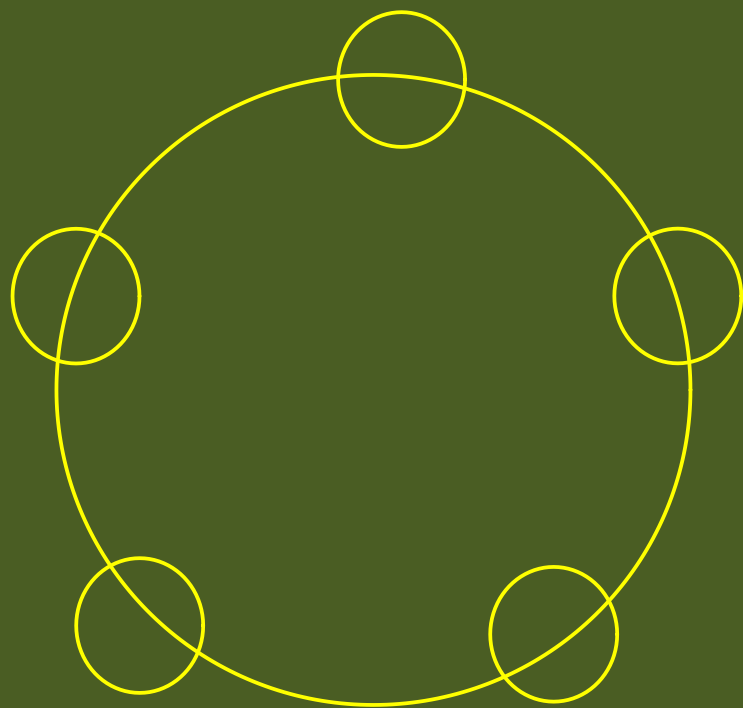
2 . **and the graph is a single piece ?**

If we can show it is true, then we have a beautiful result

A graph can be traced in a
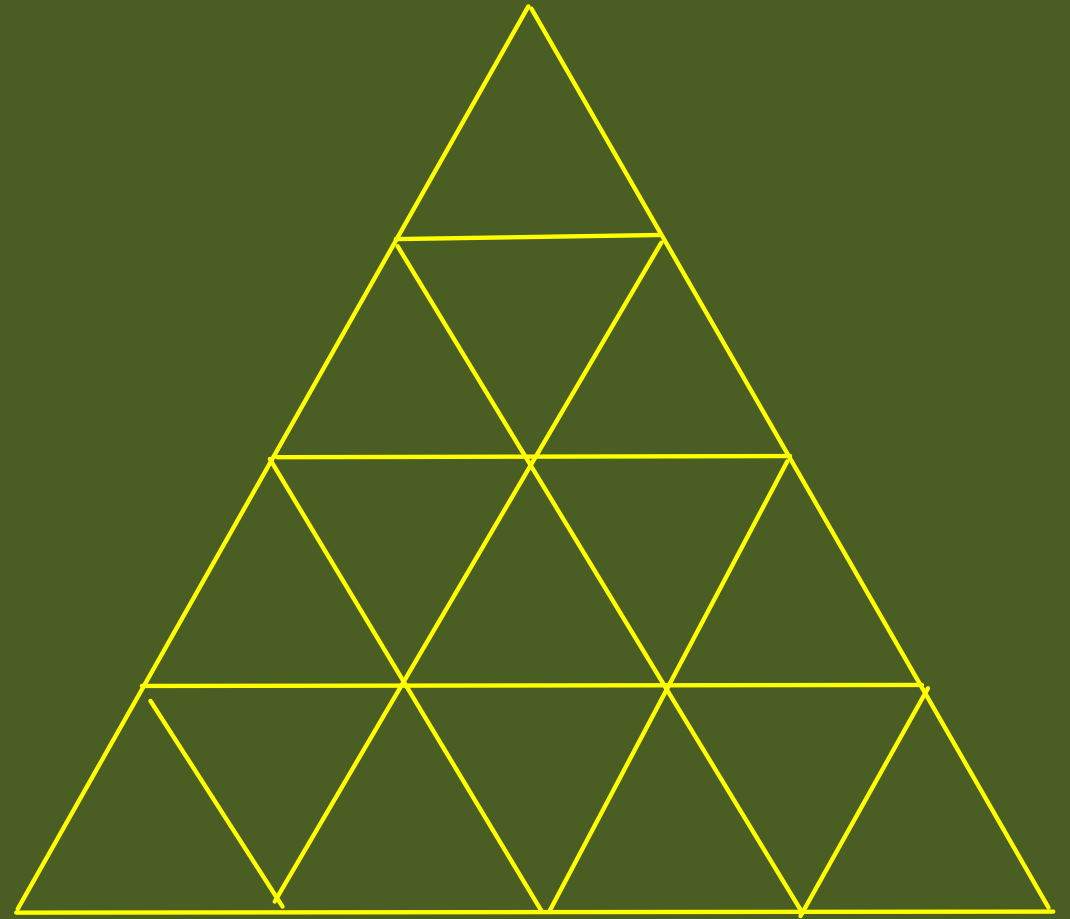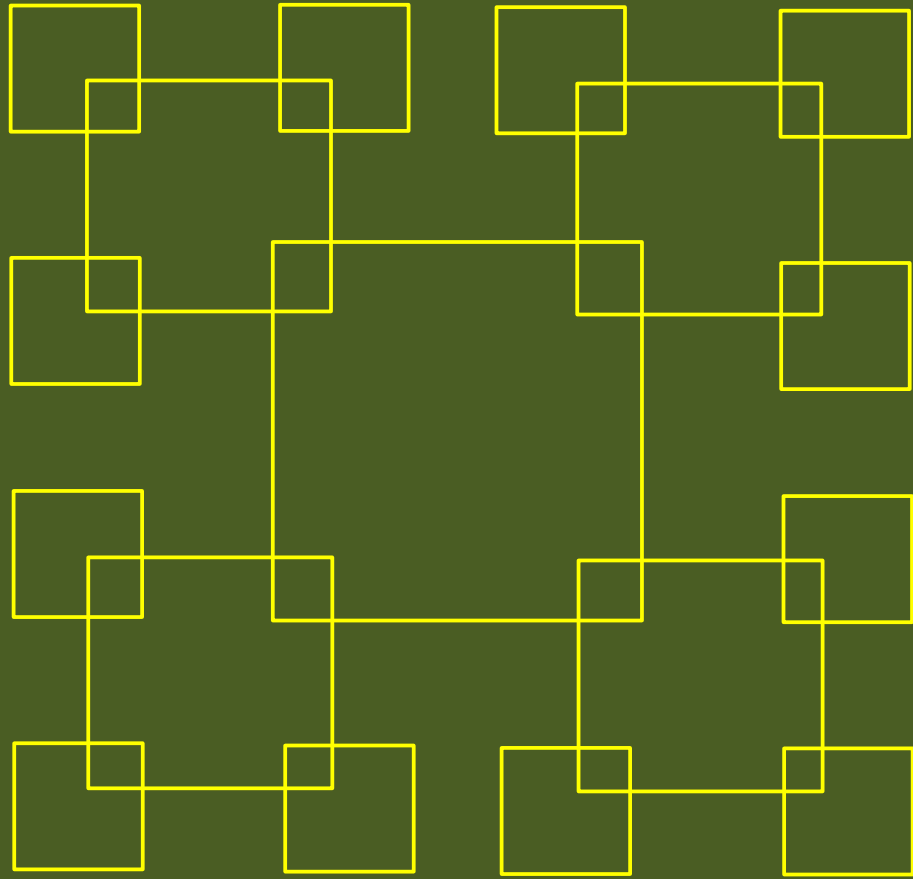**SINGLE CLOSED STROKE**

IF & ONLY IF

it is connected and
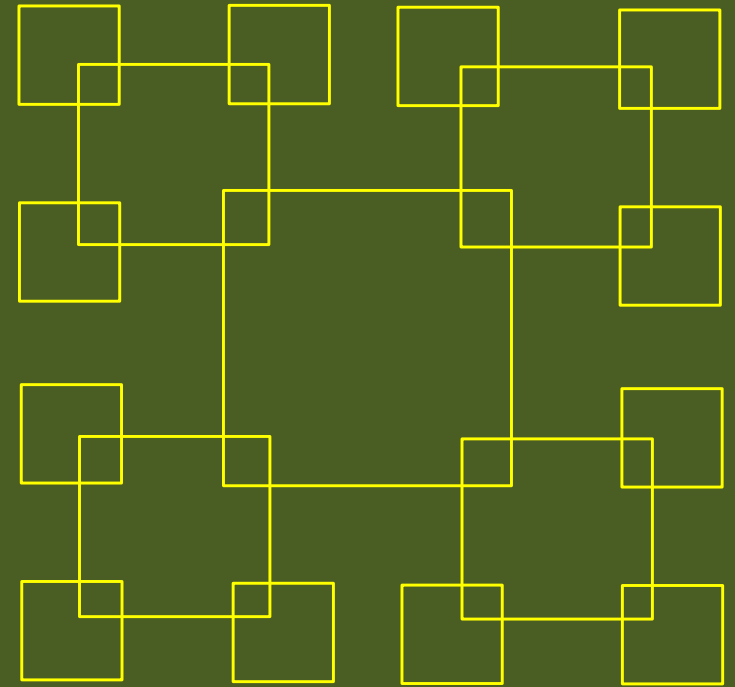**ALL DEGREES ARE EVEN**

SINGLE CLOSED STROKE

SINGLE CLOSED STROKE

A graph can be traced in a
SINGLE CLOSED STROKE

IF & ONLY IF

it is connected and
ALL DEGREES ARE EVEN

This was first proved by the famous Mathematician Leonhard Euler in 1736
in his attempt to see why the people of Konigsberg failed