

# RELATIONS

- ▶ Relationships between sets
  - ▶ Equality, subset, superset
  - ▶ Disjoint, overlapping
- ▶ Relationship between elements of a set
  - ▶ Natural numbers  $\mathbb{N}$
  - ▶ Set of triangles on a plane
  - ▶ Set of lines on a plane
- ▶ Relationship between elements of one set and elements of another set
  - ▶ Set of points on a plane and set of lines on a plane

# RELATIONS

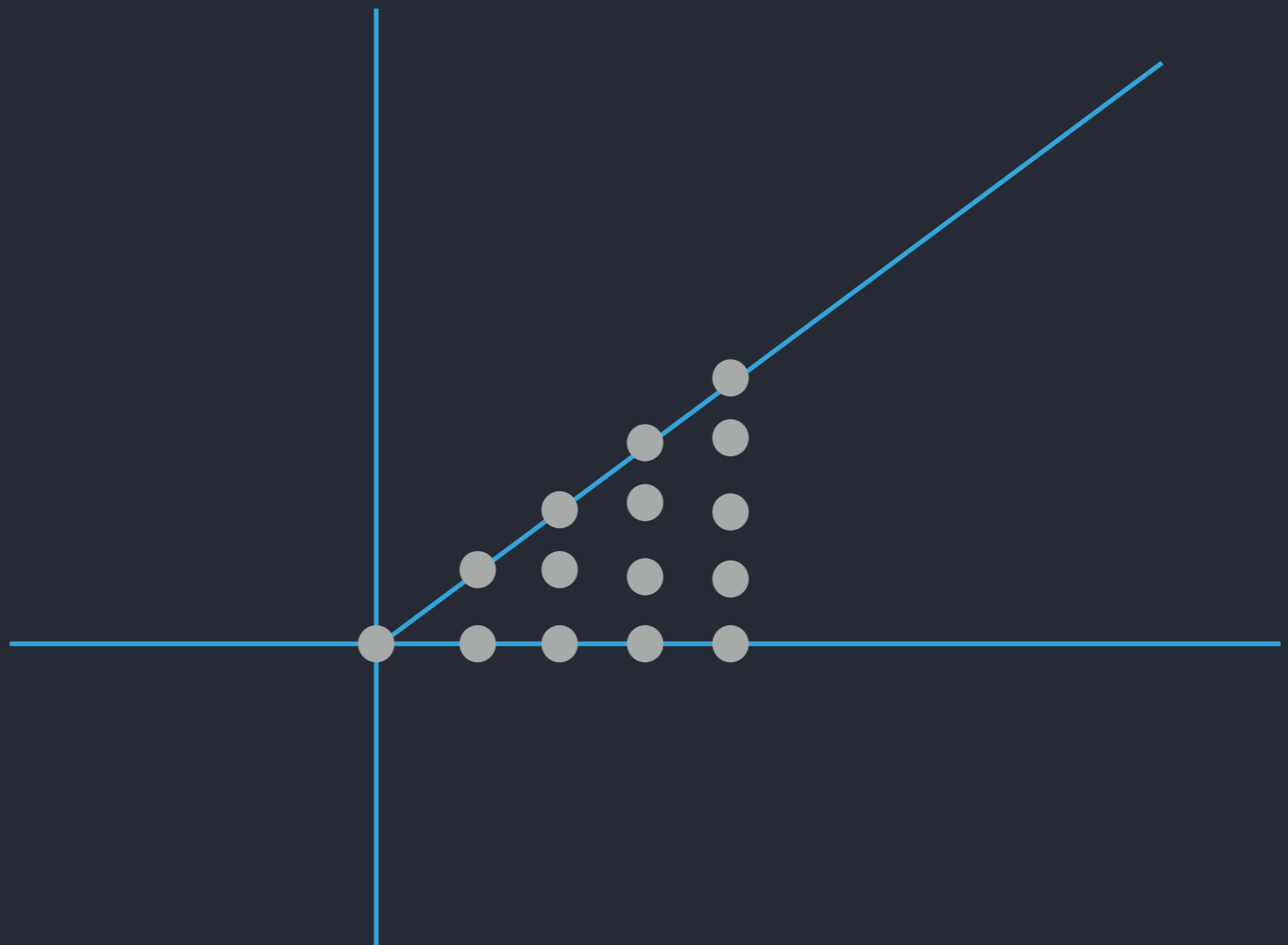
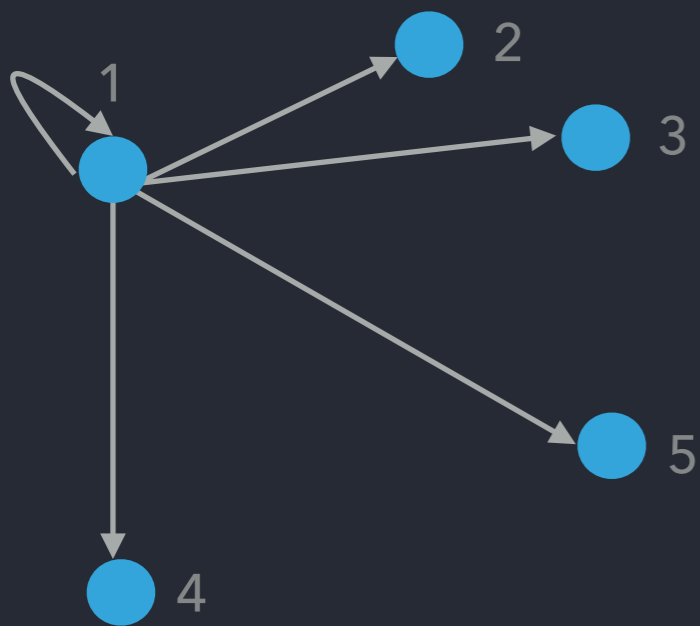
- ▶ Let  $M$  denote the set of all Palakkad Math Circle students
- ▶ Relation 1: Student  $x$  is related to  $y$  if  $x$  and  $y$  are in same school
- ▶ Relation 2: Student  $x$  is related to  $y$  if  $x$  and  $y$  are in the same class
- ▶ Relation 3: Student  $x$  is related to  $y$  if  $x$  knows  $y$
- ▶ How will you represent the set of all pairs  $x, y$  such that  $x$  is related to  $y$ ?
- ▶  $R = \{(x, y) : x, y \in M, x \text{ and } y \text{ are in same school}\}$  Note:  $R \subseteq M \times M$
- ▶ A binary relation  $R$  on a non-empty set  $A$  is a subset of  $A \times A$
- ▶ Notation:  $a R b$  denotes  $(a, b) \in R$

# RELATIONS

▶  $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a \leq b\}$

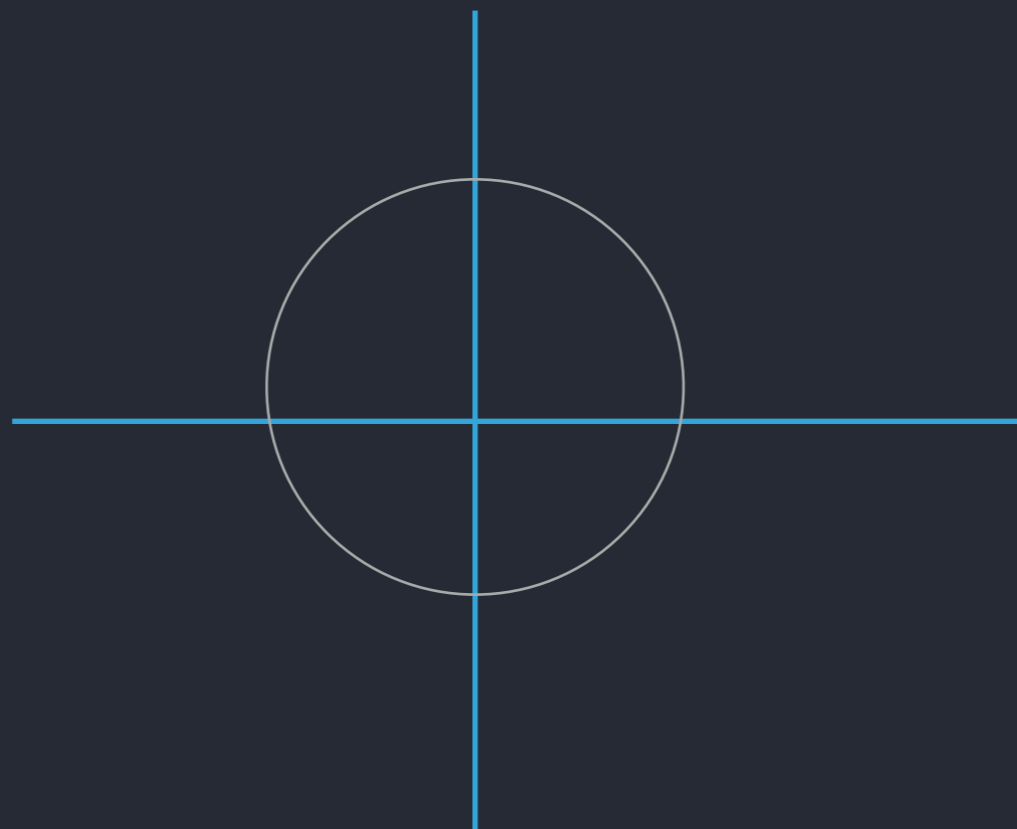
▶ Is 1 related to 2?

▶ Is 2 related to 1?



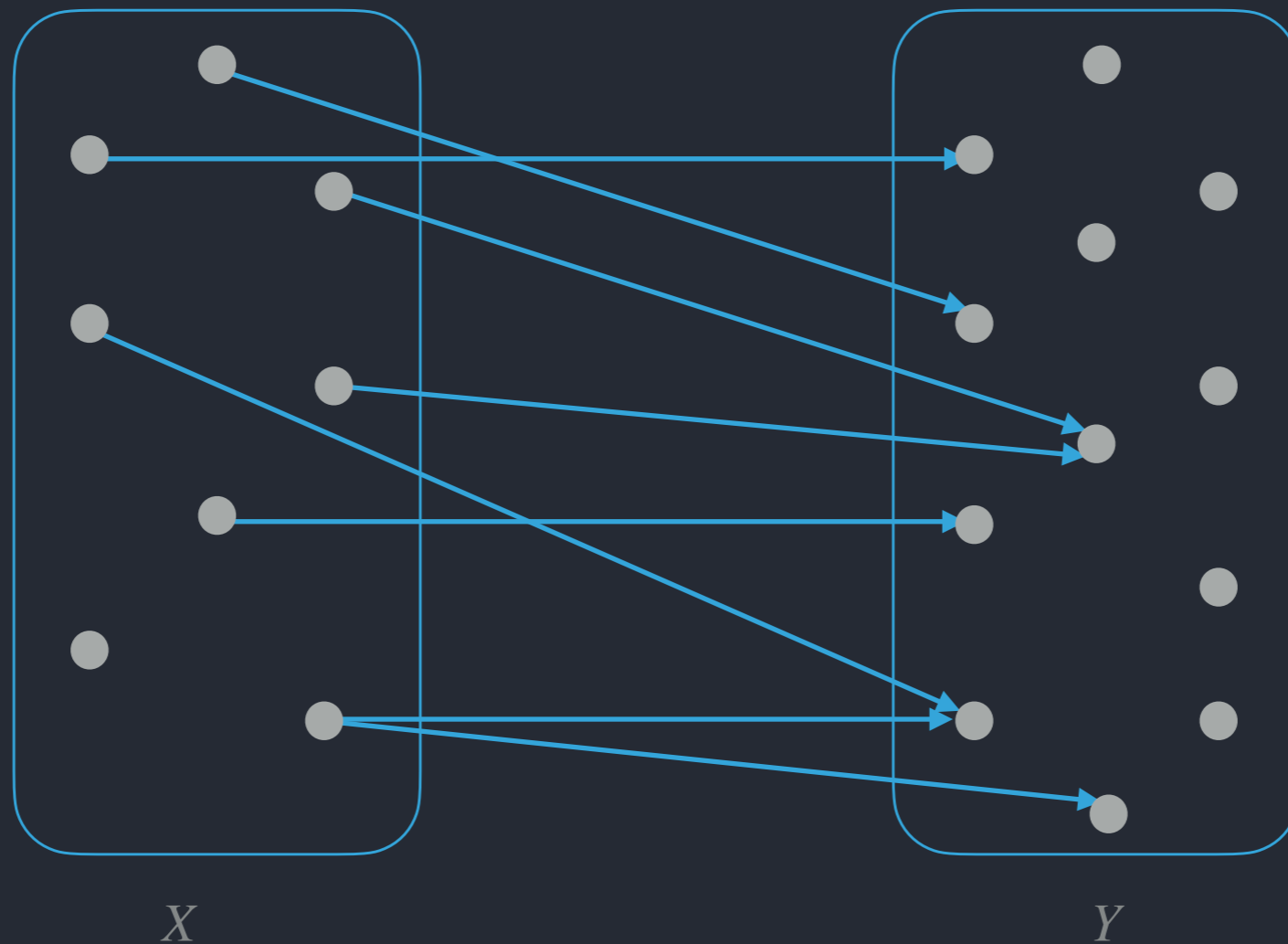
# RELATIONS

- ▶  $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 1\}$ 
  - ▶ The unit circle centred at the origin in  $\mathbb{R}^2$
  - ▶ Is 1 related to 1?
  - ▶ Find all real numbers  $y$  such that 0 is related to  $y$

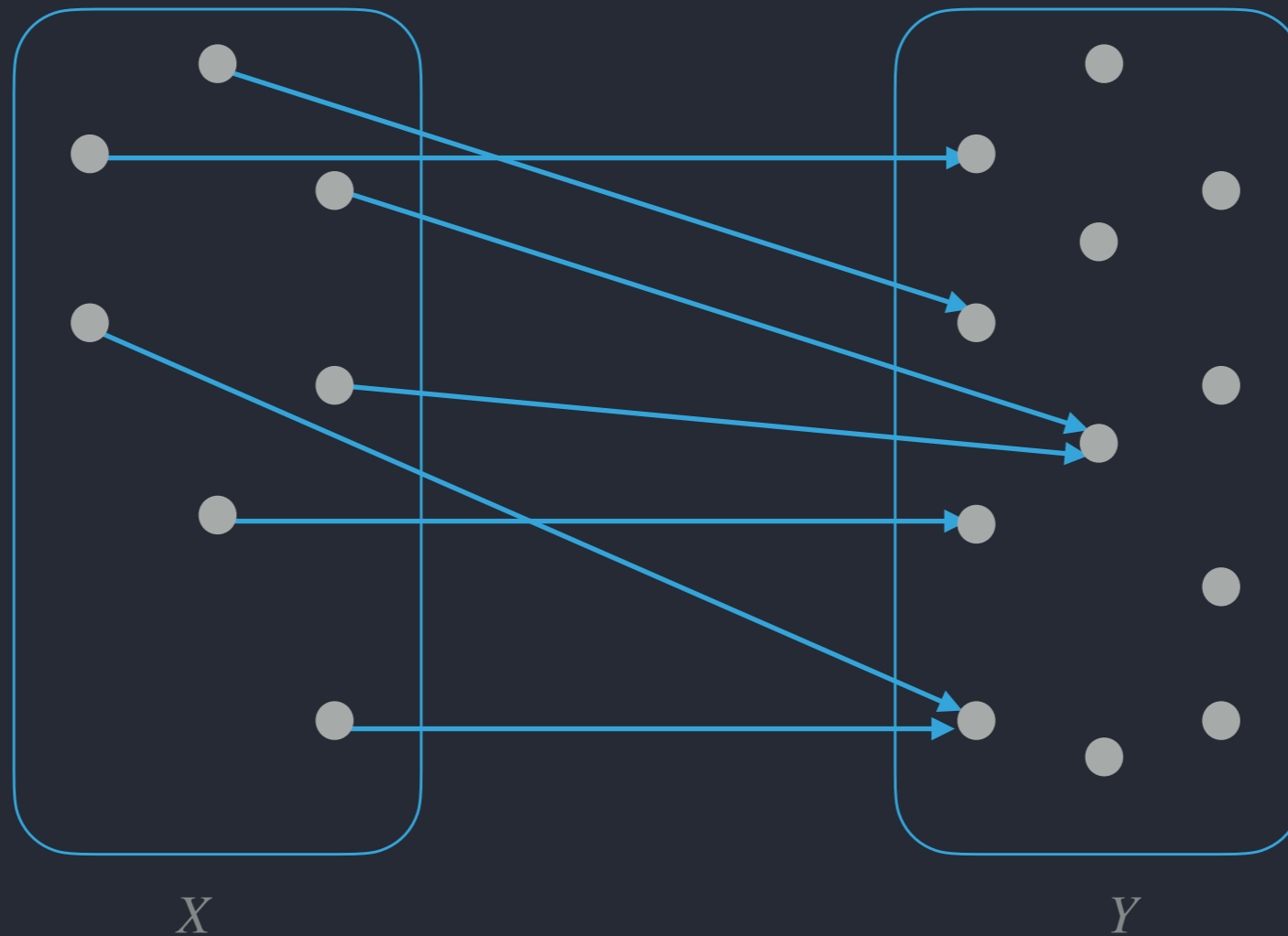


# RELATIONS

- ▶ A relation between nonempty sets  $X$  and  $Y$  is a subset  $R \subseteq X \times Y$



# FUNCTIONS



- ▶ A relation  $R$  from a set  $X$  to a set  $Y$  is called a **function** if for each element  $x \in X$  there exists a unique  $y \in Y$  such that  $(x, y) \in R$
- ▶ We define  $f: X \rightarrow Y$  by setting  $f(x) = y$  if  $(x, y) \in R$

# FUNCTIONS

- ▶ Which of the following relations are functions?
  - ▶  $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a \leq b\}$
  - ▶  $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 1\}$
  - ▶  $R = \{(x, x) : x \in X\}$  where  $X$  is a nonempty set
  - ▶  $R = \{(x, y) : x, y \in M, x \text{ and } y \text{ are in same school}\}$